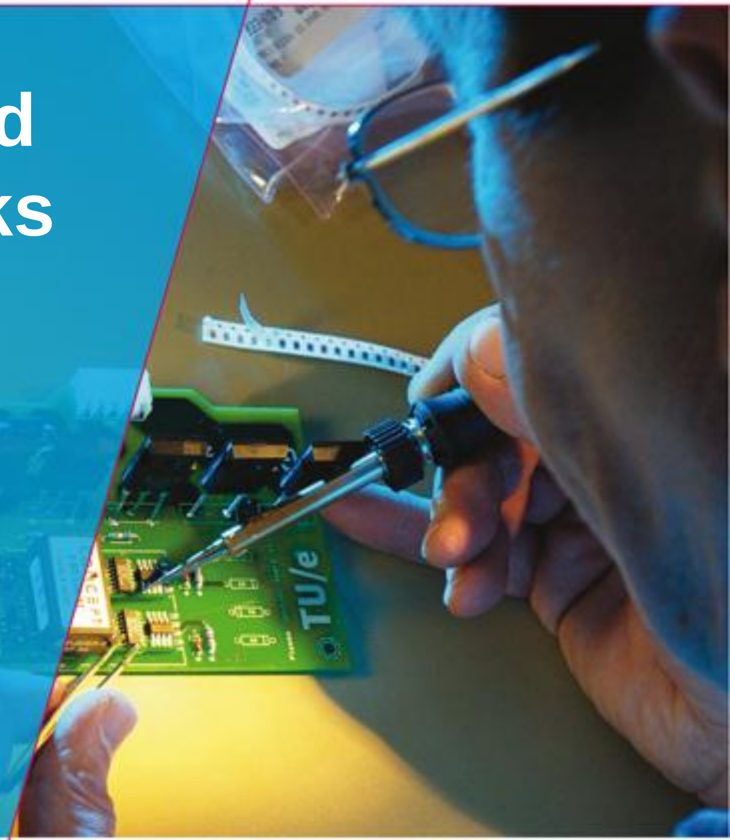
Best-case response times and jitter analysis of real-time tasks with arbitrary deadlines



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Where innovation starts

Background and motivation

- **Facts about fixed-priority scheduling (FPS):**
  - **described in standards, e.g. OSEK (Automotive);**
  - **supported by most COTS RTOS;**
  - **de facto standard in industry.**

- **Best-case analysis:**
  - **Complementary schedulability: not *too early*;**
  - **Improved *finalization jitter*;**
  - **Improved *end-to-end response times* [27].**

[27] J. Palencia Gutiérrez et al., EWRTS, 1998.

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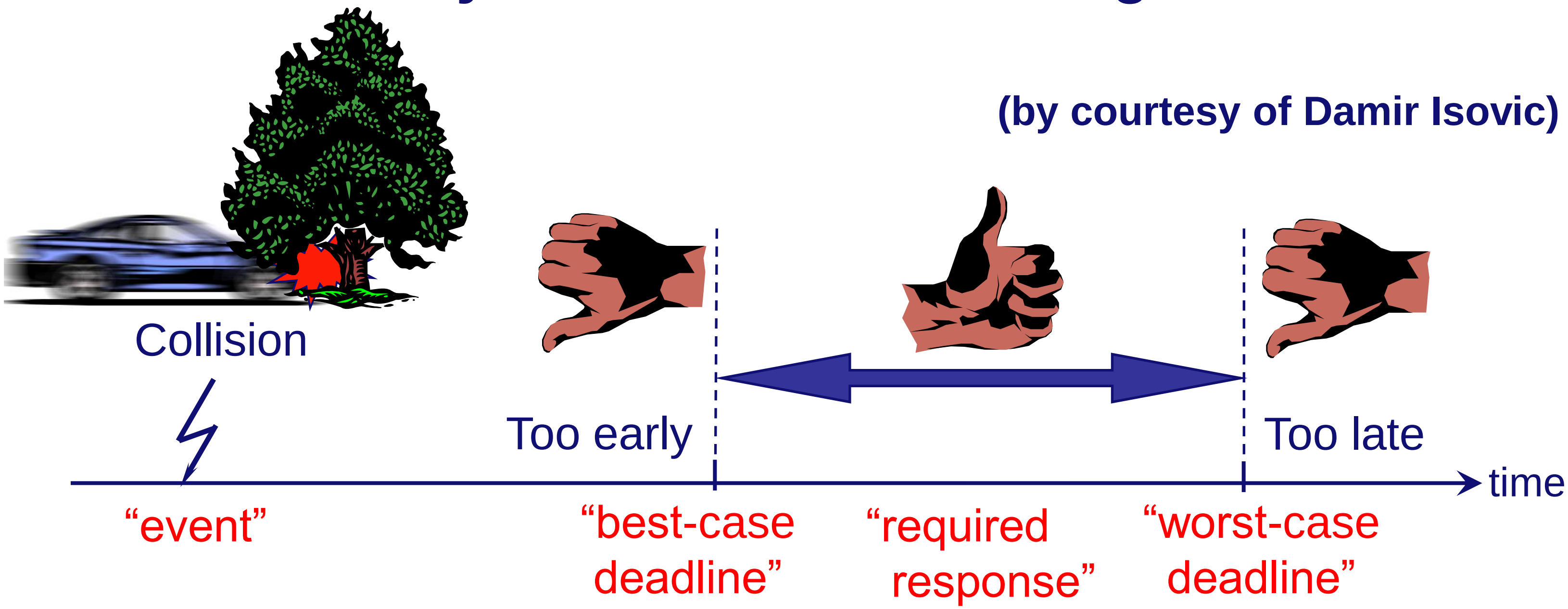
Background and motivation

- **Schedulability: inflation of an air bag**



- **Schedulability condition:**
  – all jobs of all tasks must meet their deadline constraints

$$\forall_{i,k,\varphi} \; BD_i \le R_{i,k}(\varphi) \le WD_i$$

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Background and motivation

- **State-of-research [15, 27, 29, 6]**
  - **Best-case response time and jitter analysis of independent tasks with *constrained deadlines.***

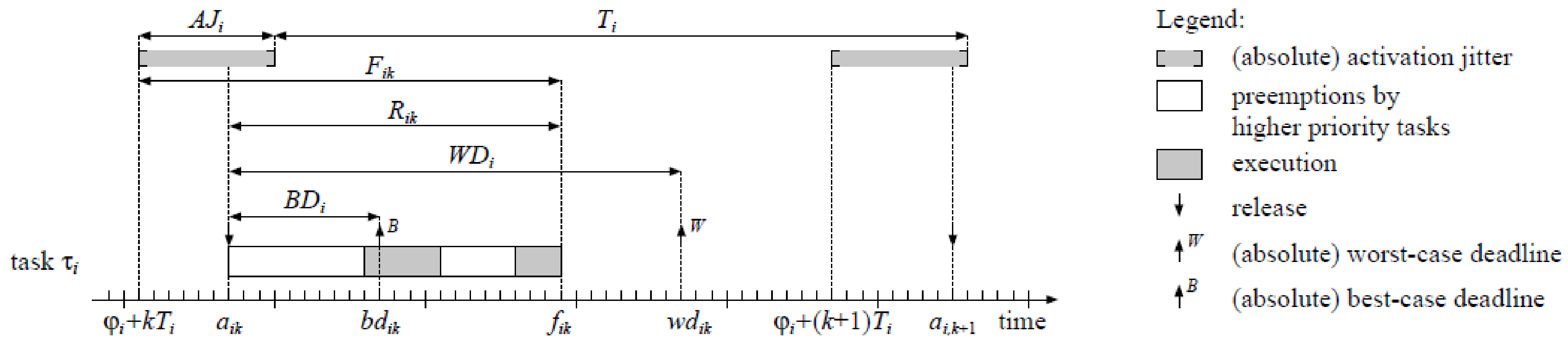
- **Goal**
  - **Best-case response time and jitter analysis of independent tasks with *arbitrary deadlines.***

[15] P. Harter, University of Colorado, 1984.
[27] J. Palencia Gutiérrez et al., EWRTS, 1998.
[29] O. Redell and M. Sanfridson, ECRTS, 2002.
[6] R. Bril, E. Steffens, and W. Verhaegh, JoS, 2004.

Overview

- Background and motivation
- Real-time scheduling model
- Witnesses of non-duality
- Response-time analysis
  - Worst-case analysis
  - Best-case analysis
- Finalization jitter
- Conclusions

Real-time scheduling model



A classical model for fixed-priority preemptive scheduling with

- **Activation jitter AJ_i;**
- **Arbitrary phasing φ_i;**
- **Arbitrary relative (worst-case) deadlines WD_i.**

Note on terminology:

- **Finalization time $F_{i,k} = f_{i,k} - (\varphi_i + k \cdot T_i)$;**
- **Response time $R_{i,k} = f_{i,k} - a_{i,k}$;**
- **Worst-case deadline $wd_{i,k} = a_{i,k} + WD_i$.**

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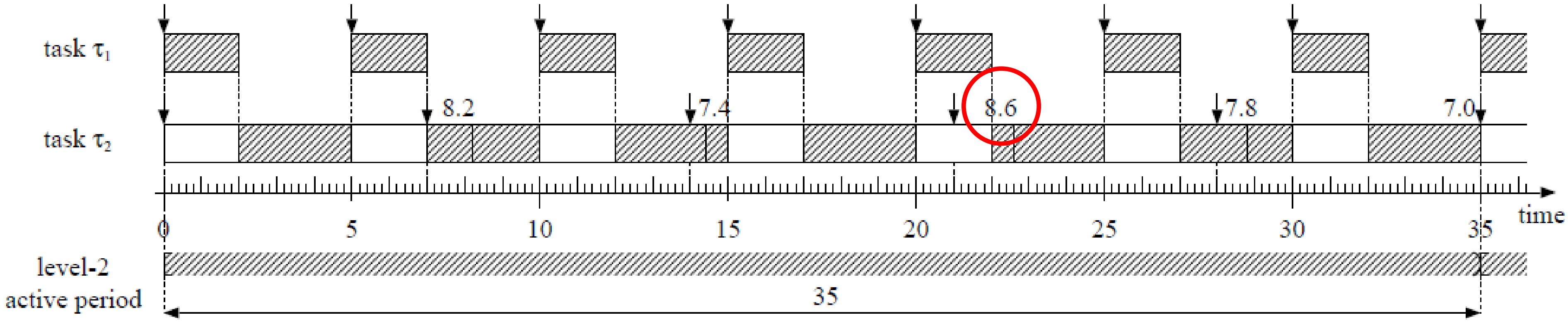
Witnesses of non-duality – example I

Task	*T*	*C*	*WD*	*WR*	*BR*
τ_1	5	2	5	2	2
τ_2	7	4.2	9.0	8.6	

Worst-case response time of task τ_2



Fact: The worst-case response time of a task τ_i with an arbitrary deadline is assumed *somewhere* in the *longest* level-i active period.

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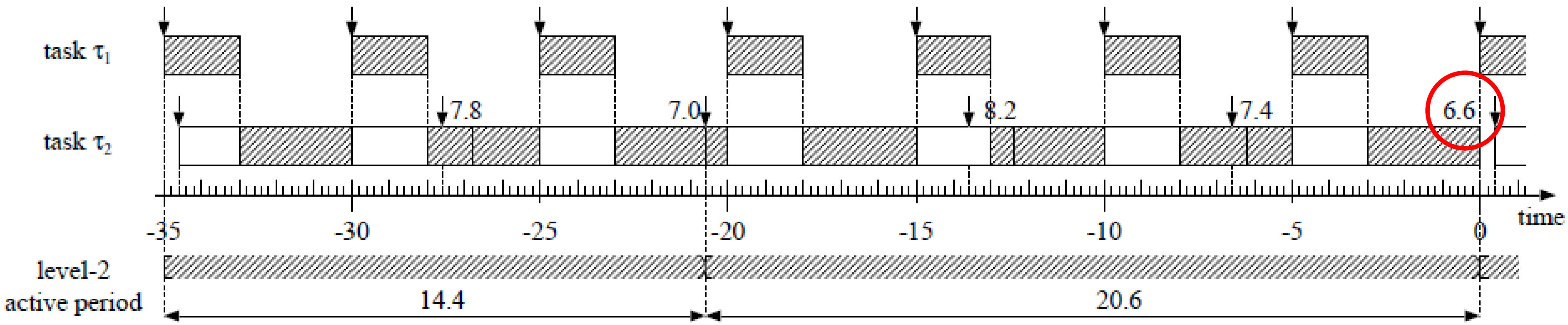
Witnesses of non-duality – example I

Task	*T*	*C*	*WD*	*WR*	*BR*
τ_1	5	2	5	2	2
τ_2	7	4.2	9.0	8.6	6.6

Best-case response time of task τ_2



Fact: The best-case response time of a task τ_i with an arbitrary deadline is not necessarily assumed in the *shortest* level-i active period.

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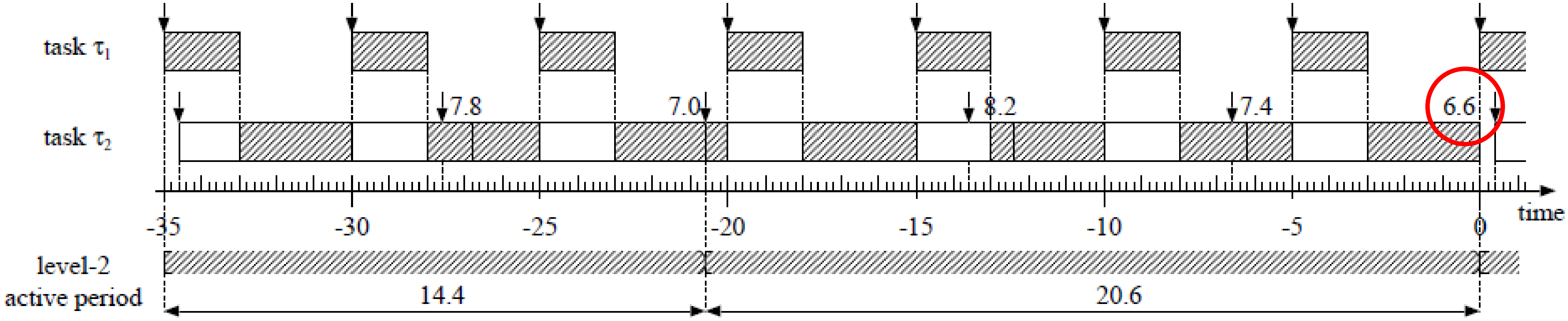
Witnesses of non-duality – example I

Task	*T*	*C*	*WD*	*WR*	*BR*
τ_1	5	2	5	2	2
τ_2	7	4.2	9.0	8.6	6.6

Best-case response time of task τ_2



Fact: The best-case response time of a task τ_i with an arbitrary deadline is always assumed by the *last job* in *a* level-i active period.

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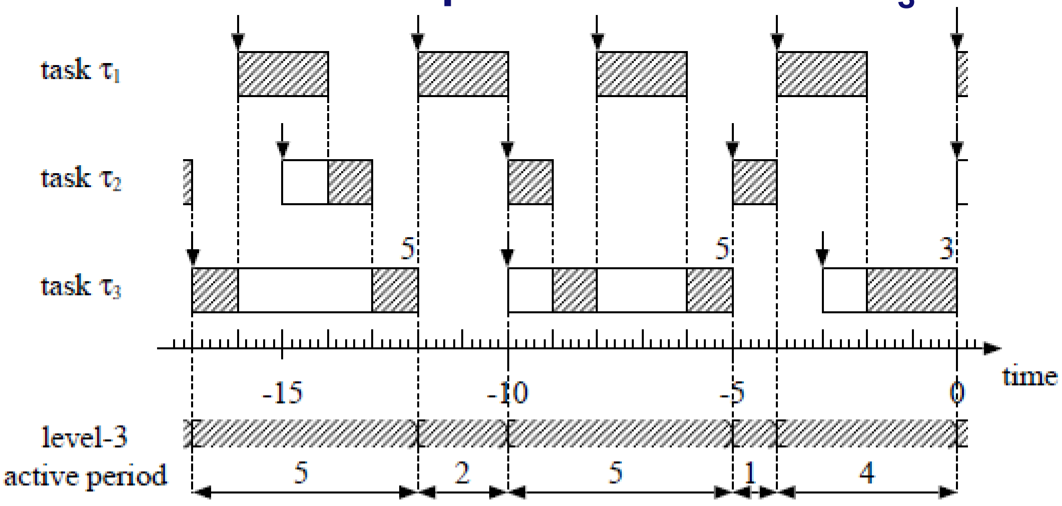
Witnesses of non-duality – example II

Task	*T*	*C*	*D*	*WR*	*BR*
τ_1	4	2	3	2	2
τ_2	5	1	4	3	1
τ_3	7	2	9	8	3

Best-case response time of task τ_3



Fact: When a job of a task τ_i with an arbitrary deadline assumes the best-case response time and cannot immediately start upon its activation, it does not necessarily experience interference of its previous job.

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Worst-case analysis

- **Based on a *critical instant***
  - **Simultaneous *release* of a task with all its higher priority tasks.**

- **Worst-case response time**
  - **Longest response time in so-called *level-i active period*.**

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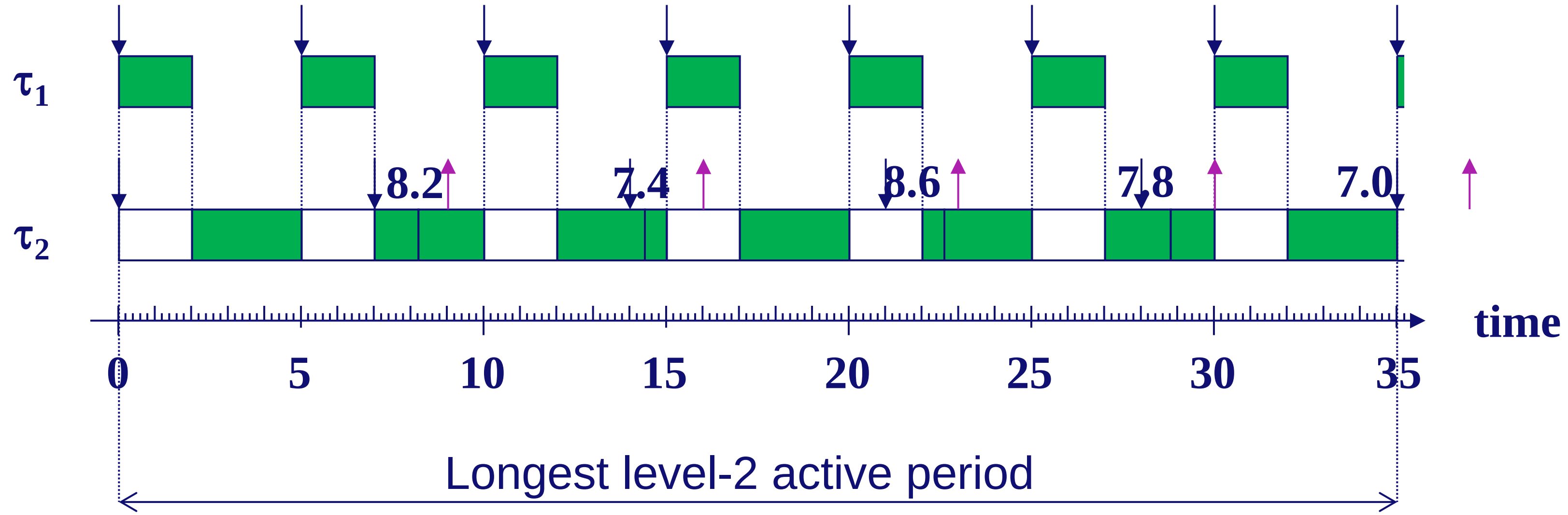
Worst-case response time of task τ_2.



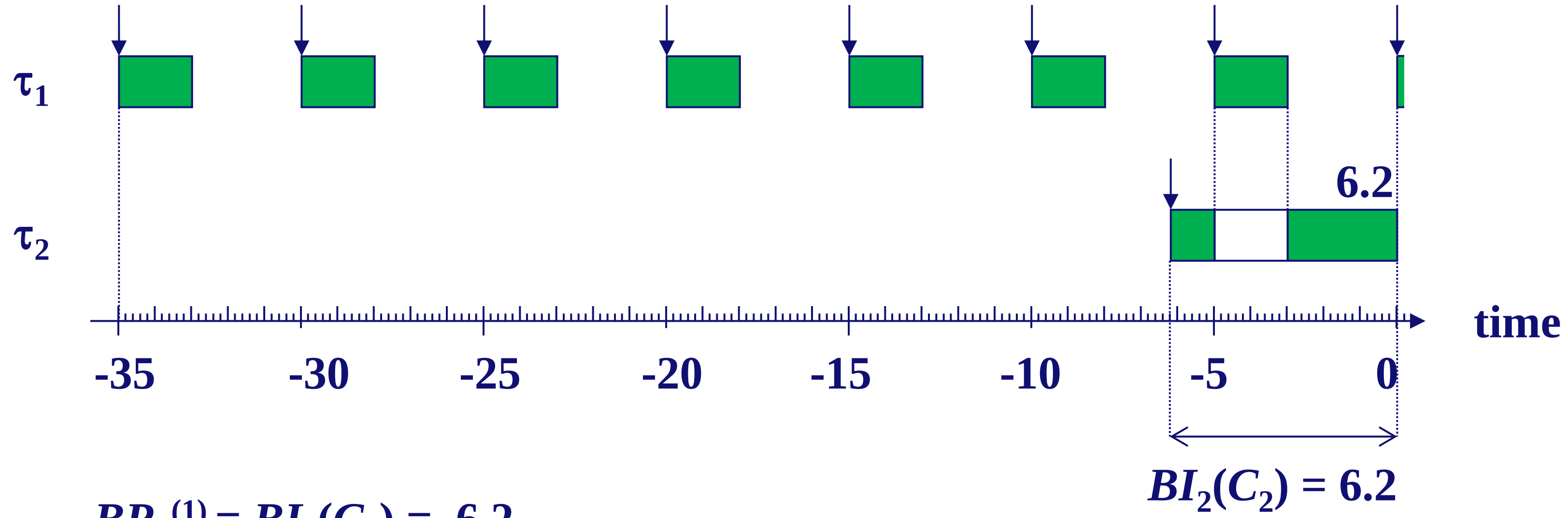
Worst-case length of level-2 active period: $WL_2 = 35.$

Number of activations of task τ_2 in WL_2 : $wl_2 = \lceil WL_2 / T_2 \rceil = \lceil 35/7 \rceil = 5$

Best-case analysis

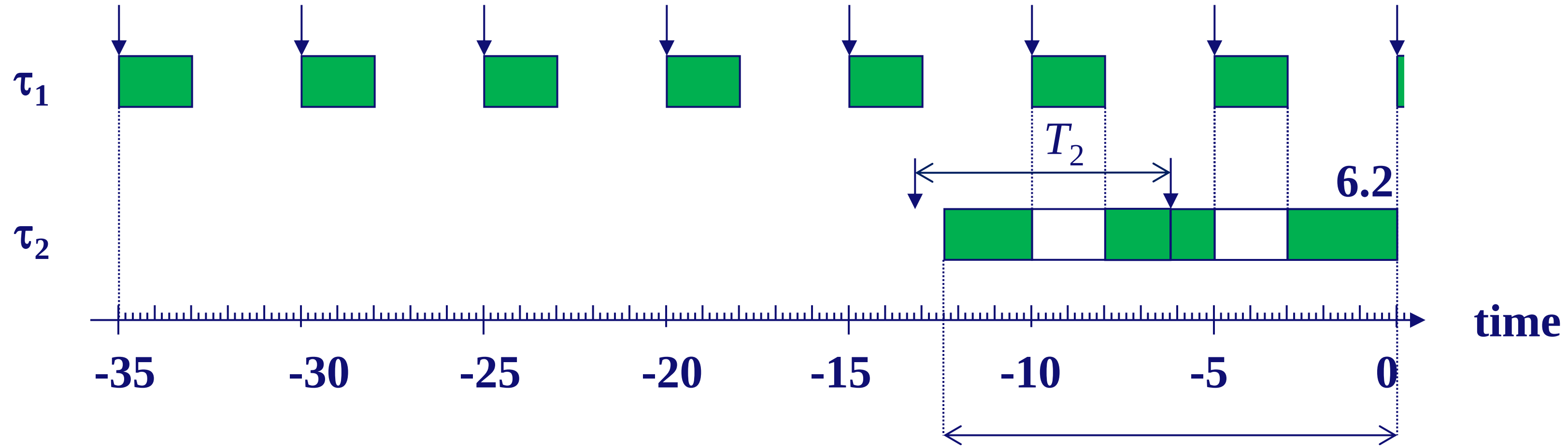
- Based on an *optimal instant*
  - *Finalization* of a task at a simultaneous release of all its higher priority tasks
- Best-case response time
  - Shortest response time of the last job in *a* level-*i* active period of a length of at most WL_i.
- Definition:
  - *The* best-case interval $BI_i(y)$ *is defined as the length of the shortest interval in which an amount of time* $y \in$ R+ *is available for the execution of a task* τ_i.
- $BI_i(y)$ is given by $BR_i(y)$ for *constrained* deadlines.

Activation of job $\tau_{2,-1}$



$BR_2^{(1)} = BI_2(C_2) = 6.2$

Activation of job $\tau_{2,-2}$



$BI_2(2 \cdot C_2) = 12.4$

$BR_2^{(1)} = BI_2(C_2) = 6.2$

$BR_2^{(2)} = \max_{1 \leq k \leq 2} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.2$

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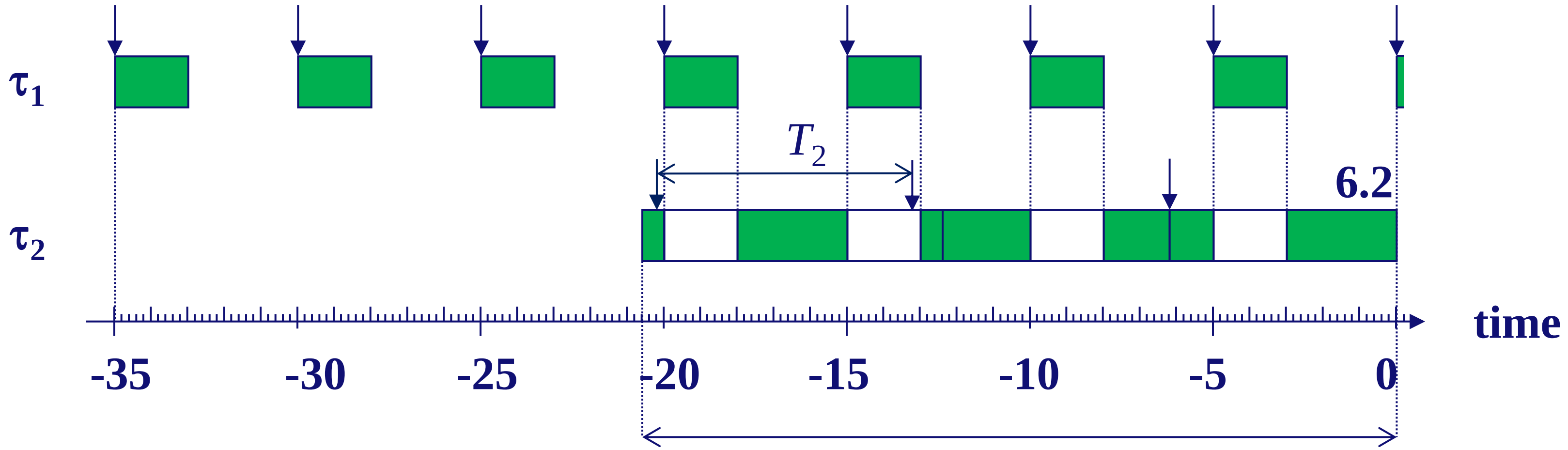
Activation of job $\tau_{2,-3}$



$BI_2(3 \cdot C_2) = 20.6$

$BR_2^{(1)} = BI_2(C_2) = 6.2$

$BR_2^{(2)} = \max_{1 \leq k \leq 2} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.2$

$BR_2^{(3)} = \max_{1 \leq k \leq 3} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$

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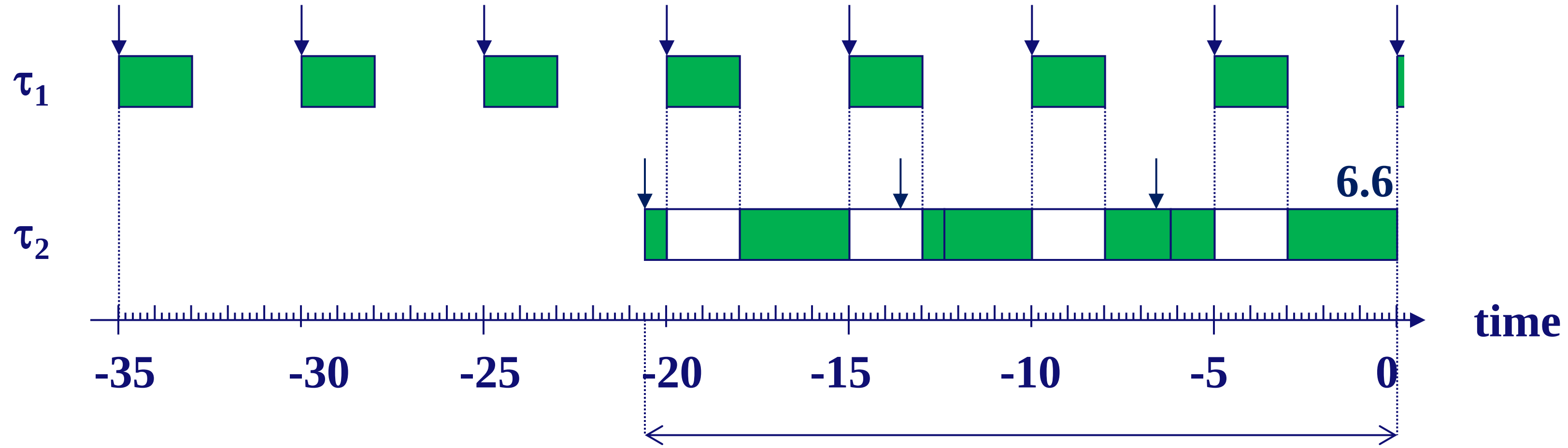
Activation of job $\tau_{2,-3}$



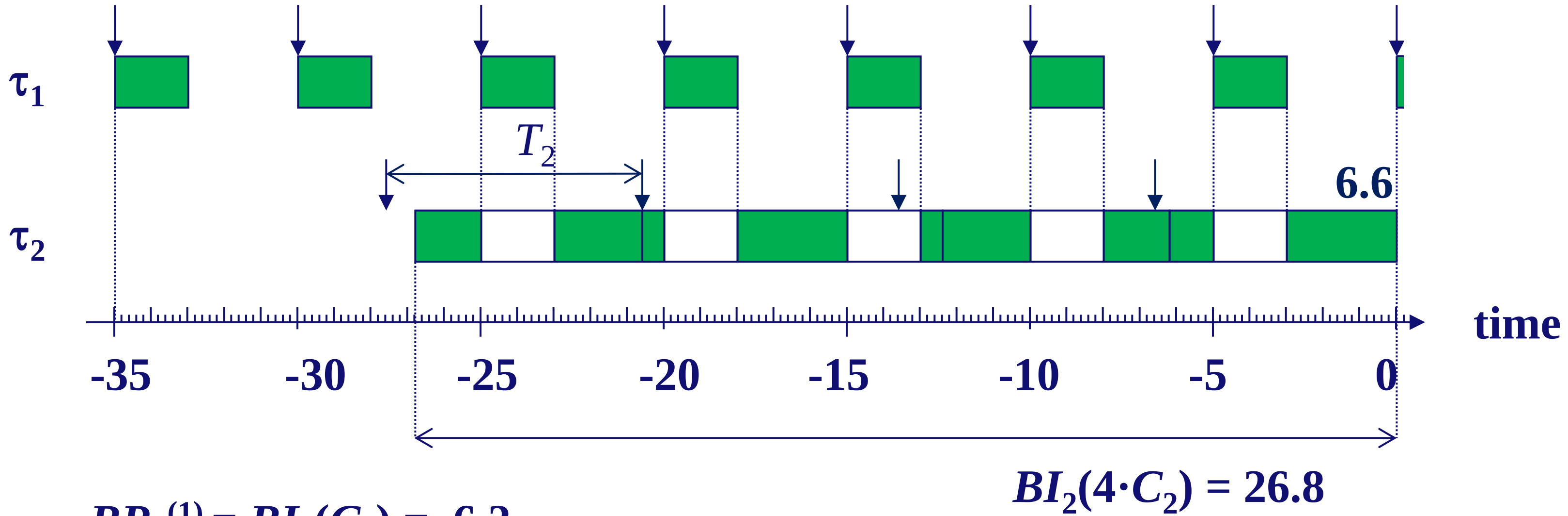
$BI_2(3 \cdot C_2) = 20.6$

$BR_2^{(1)} = BI_2(C_2) = 6.2$

$BR_2^{(2)} = \max_{1 \le k \le 2} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.2$

$BR_2^{(3)} = \max_{1 \le k \le 3} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$

Activation of job $\tau_{2,-4}$



$$BR_2^{(1)} = BI_2(C_2) = 6.2$$

$$BR_2^{(2)} = \max_{1 \le k \le 2} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.2$$

$$BR_2^{(3)} = \max_{1 \le k \le 3} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$$

$$BR_2^{(4)} = \max_{1 \le k \le 4} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$$

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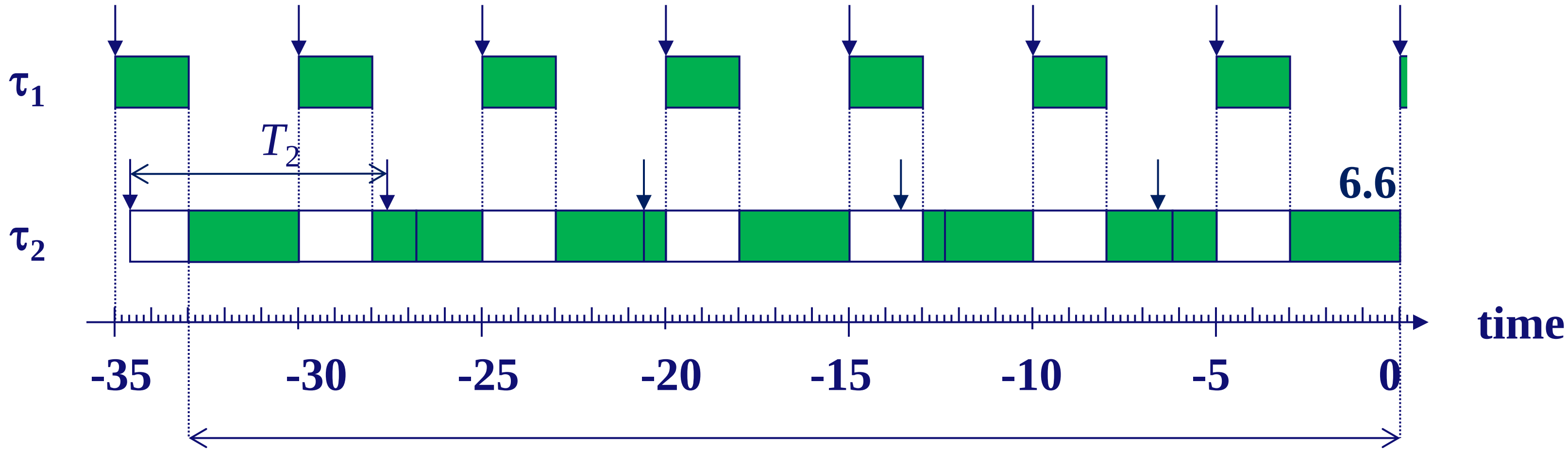
Activation of job $\tau_{2,-5}$



$BI_2(5 \cdot C_2) = 26.8$

$BR_2^{(1)} = BI_2(C_2) = 6.2$

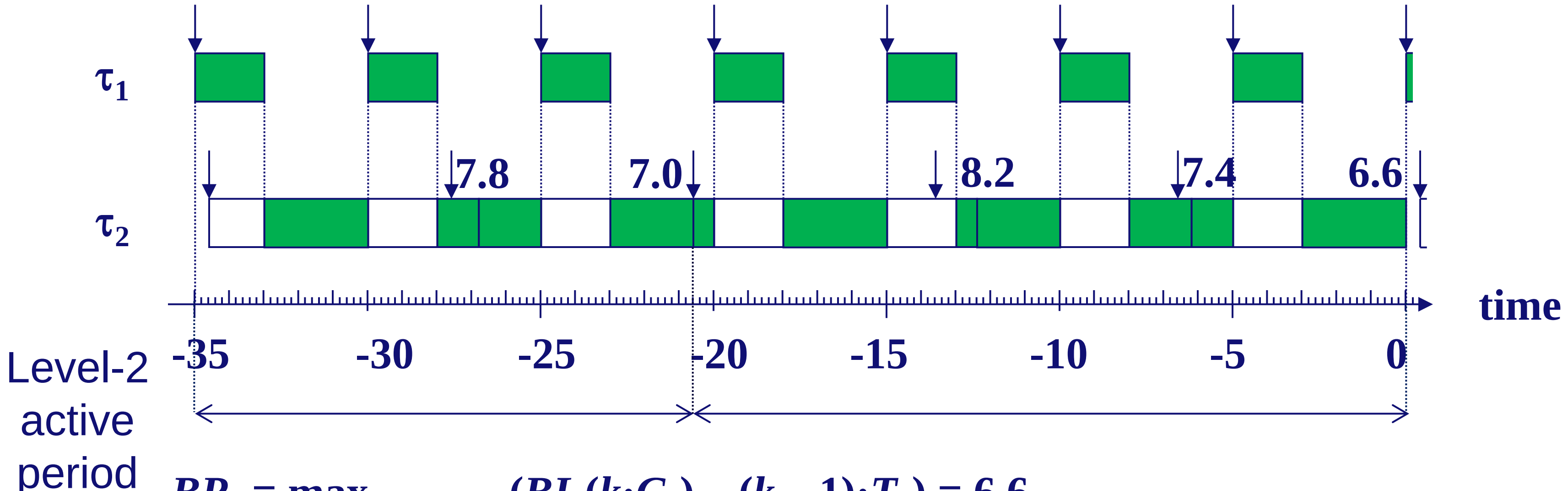
$BR_2^{(2)} = \max_{1 \leq k \leq 2}(BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.2$

$BR_2^{(3)} = \max_{1 \leq k \leq 3}(BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$

$BR_2^{(4)} = \max_{1 \leq k \leq 4}(BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$

$BR_2^{(5)} = \max_{1 \leq k \leq 5}(BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$

Best-case response time of task τ_2



$$BR_2 = \max_{1 \leq k \leq wl2} (BI_2(k \cdot C_2) - (k-1) \cdot T_2) = 6.6$$

Best-case analysis

- **Without activation jitter**
  - **Best-case response times:**
    - $BR_i = \max_{1 \leq k \leq wl_i} (BI_i(k \cdot BC_i) - (k-1)T_i);$
  - **Best-case finalization times:**
    - $BF_i = BR_i;$
- **With activation jitter**
  - **Best-case response times:**
    - $BR_i = \max_{1 \leq k \leq wl_i} \left(BI_i(k \cdot BC_i) - \begin{Bmatrix} 0 & \text{for } k = 1 \\ (k-1)T_i + AJ_i & \text{for } k > 1 \end{Bmatrix} \right);$
  - **Best-case finalization times:**
    - $BF_i = \max_{1 \leq k \leq wl_i} (BI_i(k \cdot BC_i) - (k-1)T_i);$

Finalization jitter – an example

Task	T	D	AJ	C	WR	BR	WF	BF
τ_1	4	3	0	2	2	2	2	2
τ_2	5	4	0	1	3	1	1	1
τ_3	7	9	0.6	2	8.6	2.4	8.6	3

Constrained deadlines

- $FJ_i \leq AJ_i + WR_i - BR_i$; [6]

$\check{BR}_3 = 2 \Rightarrow FJ_3 = 0.6 + 8.6 - 2 = 7.2$

Arbitrary deadlines

- $FJ_i \leq WF_i - BR_i$; [17]

$\check{BR}_3 = 2 \Rightarrow FJ_3 = 8.6 - 2 = 6.6$

$BR_3 = 2.4 \Rightarrow FJ_3 = 8.6 - 2.4 = 6.2$

- $FJ_i \leq WF_i - BF_i$; [0]

$FJ_3 = 8.6 - 3 = 5.4$

[6] R. Bril, E. Steffens, and W. Verhaegh, JoS, 2004.
[17] R. Henia, R. Racu, and R. Ernst, IPDPS, 2007.
[0] This paper/presentation.

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Contributions

- **Witnesses of non-duality**
- **Best-case analysis for arbitrary deadlines and jitter**
  - **best-case response times;**
  - **best-case finalization times.**
- **Improvements:**
  - **finalization jitter;**
  - **end-to-end response times.**
- **Illustrated by means of examples.**
- **“Basis” for further extensions.**

Acknowledgements

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  - **Liliana Cucu-Grosjean – INRIA Nancy-Grand Est**
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- **Feedback:**
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