Formal Approach to Guard Time Optimization for TDMA

Oday Jubran Bernd Westphal

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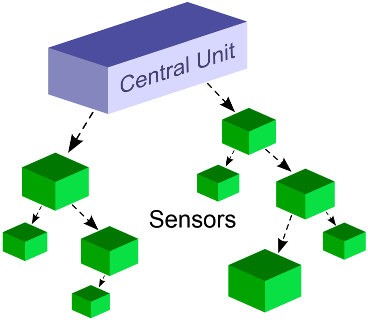
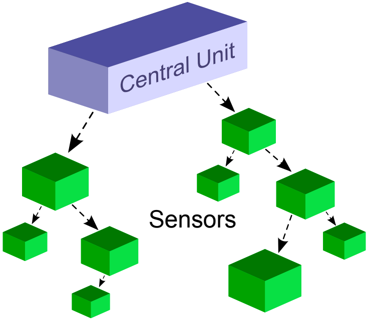


- A network with hierarchical structure, e.g. Wireless Fire Alarm System



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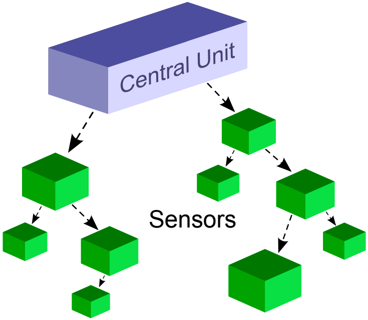
- One shared communication medium



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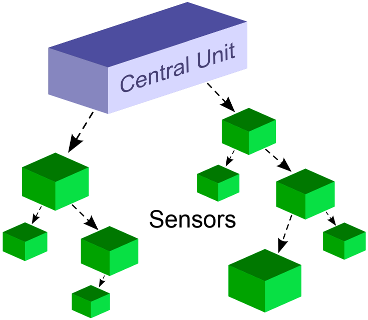
- Time Division Multiple Access (TDMA) Protocol



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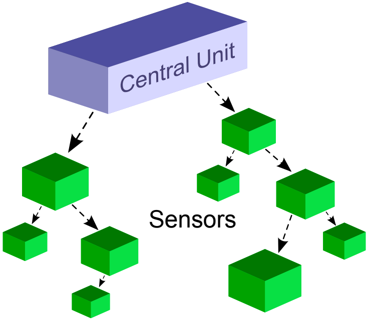

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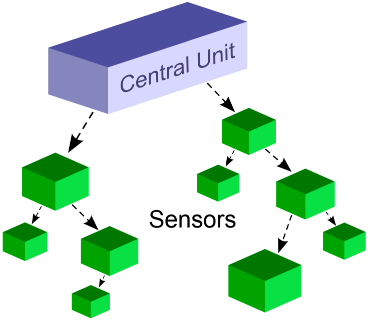

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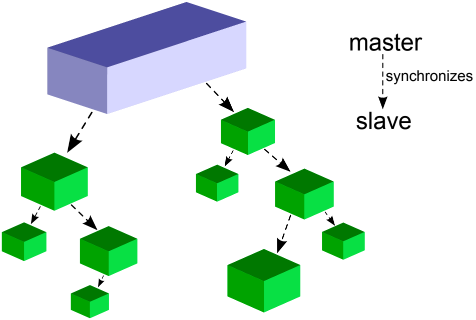
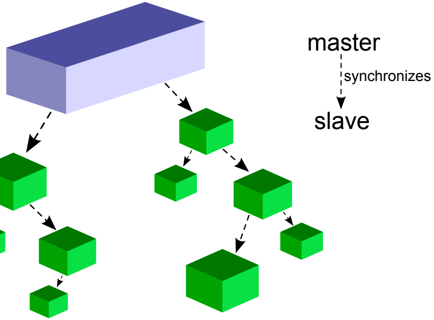
- Requirement: Collision and loss free system



- Hierarchical clock synchronization



- Hierarchical clock synchronization
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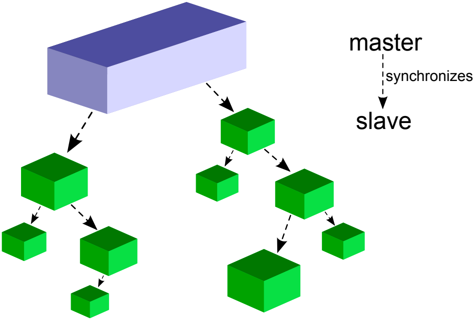

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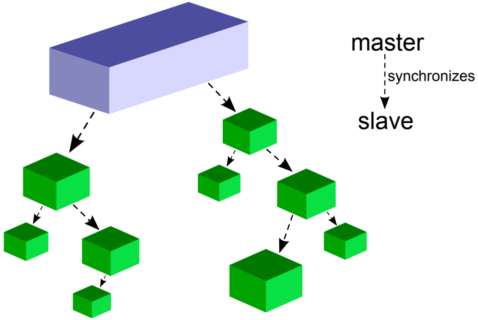

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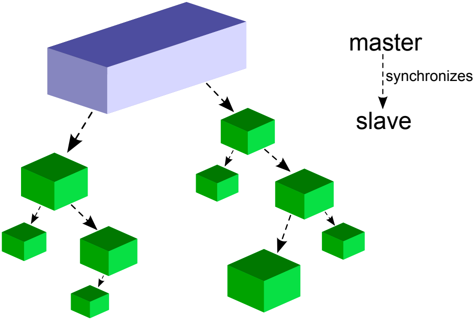

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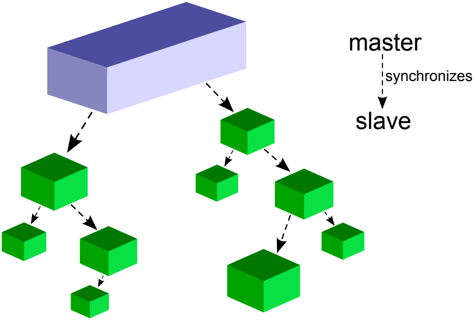

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- Our Approach: Formal guard time optimization

Outline

Formal Model

- Topology, Evolution
- Clock Drift, Collision and Loss
- TDMA, Scheduled and Synchronized Evolution
- Guard Time

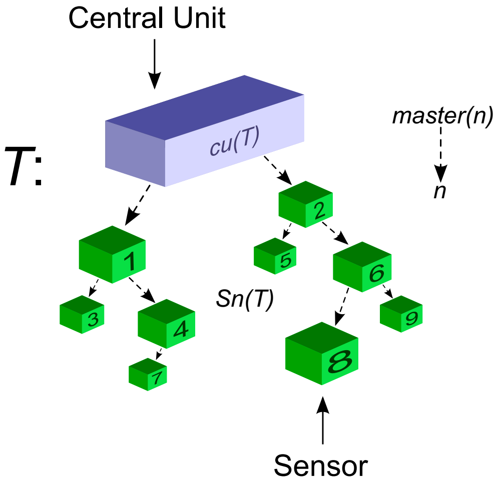
Guard Time Optimization

- Worst Case Assignment
- Best Case Assignment

Conclusion

Formal Model

Evolutions over a Topology



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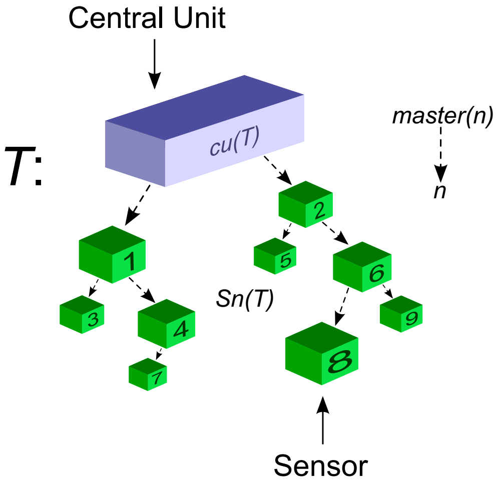

Evolution

An interpretation $\mathcal{I}$ of

1. clk_n : Time($= \mathbb{R}_0^+$),
2. $send_n : \mathbb{B}$,
3. $listen_n : \mathbb{B}$,

for each $n \in N$.

Assumptions:

- For each $t \in$ Time, $clk^{\mathcal{I}}_{cu(T)}(t) = t$.
- $clk^{\mathcal{I}}_n(0) = 0$.

Clock Drift

- The clock speed of node n in $\mathcal{I}$ is:

$$\varphi_n^{\mathcal{I}} = \frac{\partial}{\partial t}\, clk_n^{\mathcal{I}}(t).$$

- The clock drift of n in $\mathcal{I}$ at time t is:

$$\varrho_n^{\mathcal{I}}(t) = clk_n^{\mathcal{I}}(t) - clk_{cu(T)}^{\mathcal{I}}(t).$$

- The drift rate of n in $\mathcal{I}$ is:

$$\Delta_n^{\mathcal{I}} = \frac{\partial}{\partial t}\, \varrho_n^{\mathcal{I}}(t).$$

A maximum drift rate $\Delta^{max} \in \mathbb{R}_0^+$ is a least upper bound on $|\Delta_n^{\mathcal{I}}|$ for any node n in $\mathcal{I}$.

Message Collision and Loss / TDMA

In evolution $\mathcal{I}$, there is

- message collision at time t between two sensors n_1, n_2 iff

$$send^{\mathcal{I}}_{n_1}(t) \wedge send^{\mathcal{I}}_{n_2}(t),$$

- message loss for a sensor n at time t iff

$$send^{\mathcal{I}}_{n}(t) \wedge \neg listen^{\mathcal{I}}_{master(n)}(t).$$

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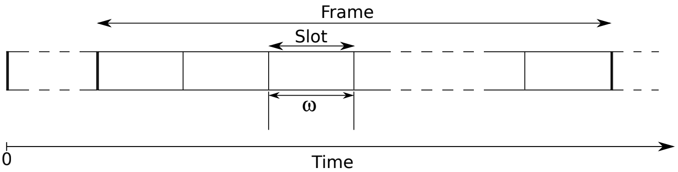
TDMA: Partitioning of time into frames and slots



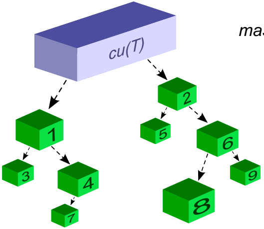
Scheduled Evolution

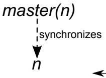
- Bijective assignment of slots of a frame to sensors
- A sensor sends only during its assigned slot.
- $master(n)$ listens during the slot assigned to n

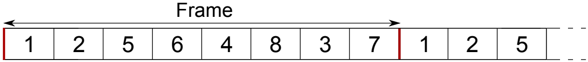
Synchronized Evolution

Each sensor n has at least one point $t \in \text{Time}$ in each of its slots, where

$$clk_n^{\mathcal{I}}(t) = clk_{master(n)}^{\mathcal{I}}(t).$$

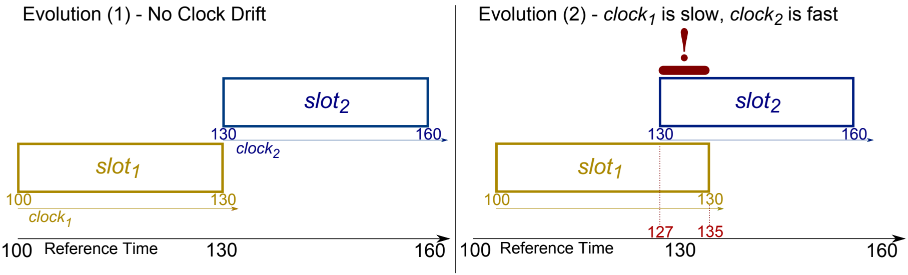






Message Collision Due to Clock Drift

Clocks may still drift.

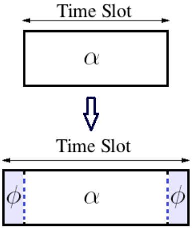


Message Collision and/or Message Loss!

Guard Time

Use Guard Time ϕ

During ϕ, no send, but only listen



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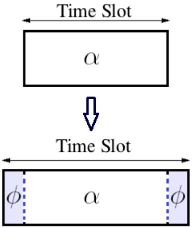

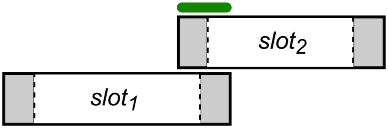

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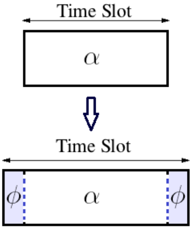

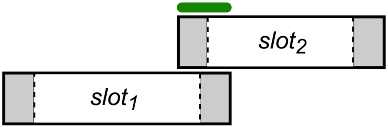

Drawback: Guard time requires extra time and energy

Formal Guard Time Optimization

Safe Guard Time

Evolution with safe guard time exhibits neither message collision nor loss

Optimal guard time ϕ_{opt} is the minimal safe guard time

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Optimal guard time ϕ_{opt} is the minimal safe guard time

Theorem 1

For a scheduled evolution $\mathcal{I}$ over a topology T:

1. Guard time ϕ is safe if

$$\forall\, n \in Sn(T), t \in \mathrm{Time} \bullet |\varrho_n^{\mathcal{I}}(t)| \leq \frac{\phi}{2}.$$

2. Guard time ϕ is optimal if it is safe and

$$\exists\, n \in Sn(T), t \in \mathrm{Time} \bullet |\varrho_n^{\mathcal{I}}(t)| = \frac{\phi}{2}.$$

$\varrho_n^{\mathcal{I}}(t)$: Clock drift of n in $\mathcal{I}$ at time t

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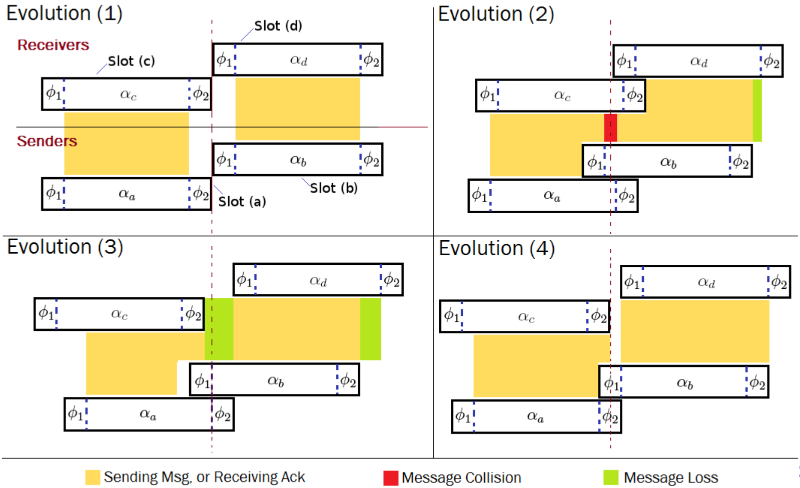
Safe Guard Time



Slot Assignment

For trees of depth $d > 1$:

The optimal guard time value depends on the slot assignment order.

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Forward Distance

- The forward distance between two sensors n_1, n_2 is the number of slots between a slot assigned to n_1 and the next slot assigned to n_2
- Approach:
  - Sum the forward distances between nodes along each path in T.
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Worst Case Assignment: The assignment that obtains the largest maximum sum of forward distances, yielding the maximal ϕ_{opt}.

Best Case Assignment: The assignment that obtains the least maximum sum of forward distances, yielding the minimal ϕ_{opt}.

Worst Case Assignment

Lemma 1 (Worst Case Assignment)

There exists in T a path

$$n_0 = cu(T), \ldots, n_{d-1}, n_d$$

such that

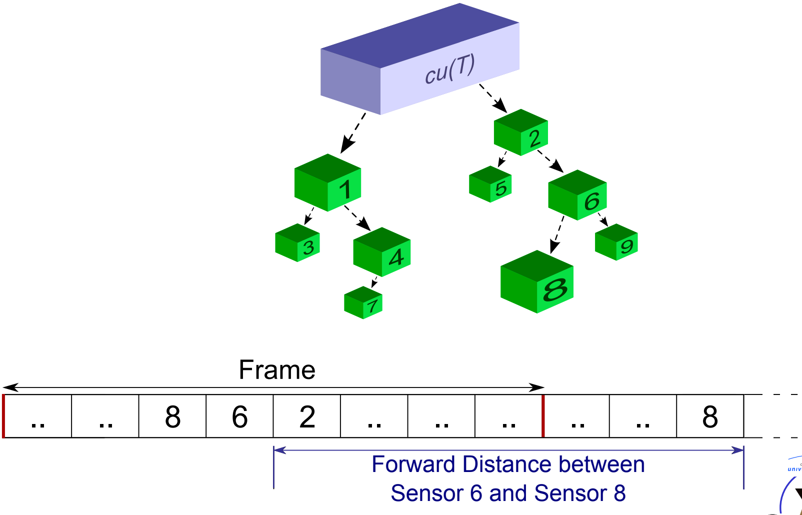
- d is the tree depth,
- the sensors of the path are assigned reverse adjacent slots.

The maximum sum of forward distances between nodes on any path is:

$$(d-1)(k-1),$$

where $k = |Sn(T)|$.

Worst Case Example



ϕ_{opt} for the Worst Case

Theorem 2 (ϕ_{opt} for the Worst Case)

For the worst case assignment, there exists an optimal guard time iff

$$\Delta^{max} < \frac{1}{4(d(k-1)+2)},$$

and is given by

$$\phi_{opt} = \alpha \cdot \frac{2(d(k-1)+2) \cdot \Delta^{max}}{1 - 4(d(k-1)+2) \cdot \Delta^{max}},$$

where

- $k = |Sn(T)|$,
- α: Slot length excluding guard time,
- Δ^{max}: Maximum drift rate.

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Best Case Assignment

Lemma 2 (Best Case Assignment)

For each subtree rooted by a sensor at depth 1:

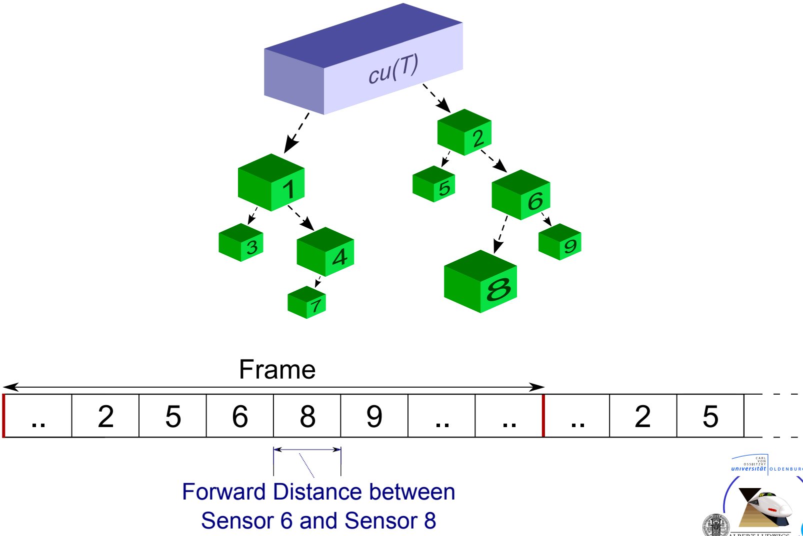
- The subtree sensors are assigned adjacent slots.
- Each slave is assigned a slot that is after the slot assigned to its master.

The maximum sum of forward distances between nodes on any path is:

$$K - 1,$$

where K is the size of the largest subtree rooted by a sensor.

Best Case Example



ϕ_{opt} for the Best Case

Theorem 3 (ϕ_{opt} for the Best Case)

For the best case assignment, there exists an optimal guard time iff

$$\Delta^{max} < \frac{1}{4(K+k)},$$

and is given by

$$\phi_{opt} = \alpha \cdot \frac{2(K+k) \cdot \Delta^{max}}{1 - 4(K+k) \cdot \Delta^{max}},$$

where

- K: The largest subtree rooted by a sensor,
- k: Number of slots per frame,
- α: Slot length excluding guard time,
- Δ^{max}: Maximum drift rate.

Conclusion

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Guard time can be computed as being

- **optimal** for a given assignment, worst case, and best case assignments,
- **safe** for an arbitrary assignment, by choosing the optimal one for the worst case assignment.

Safe guard time can be set beforehand, while restricting the network size and the slot assignment order.

A bounded message loss due to signal issues can be tolerated by increasing the guard time length.

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Thank You for Your Attention!