Continuity for Network Calculus

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ONERA – The French Aerospace Lab

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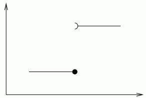
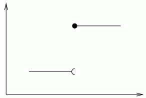
retour sur innovation

What is the problem?

- Network calculus:
  - a theory to compute memory and delay bounds in networks
  - used to certify A380 backbone
- Network calculus handle “arrival curves” functions
- Packet arrival create discontinuities
  - Some papers assume left-continuous functions (start of busy period is well defined)



  - Some papers assume right-continuous functions (departure time of packet is well defined)



- Everybody claim that is does not make any difference and that is does not require to pass time on it
- We prove it was right

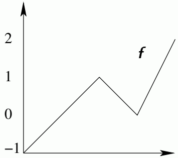
Outline

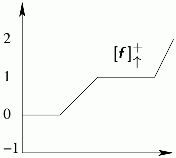
- Based on the (min,plus) dioid
  - Two basic operators: $\wedge$, minimum and $+$, sum
  - Some associated operators: convolution $*$, sub-additive closure $\cdot^*$, non-negative non-decreasing closure $[\cdot]_{\uparrow}^{+}$

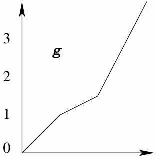
$$(f * g)(t) = \inf_{0 \leq s \leq t} f(t-s) + g(s)$$

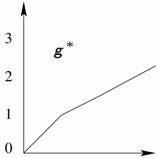
$$f^* = \delta_0 \wedge f \wedge (f * f) \wedge (f * f * f) \wedge \cdots$$

$$[f]_{\uparrow}^{+}(t) = \sup_{0 \leq s \leq t} \{\max(f(s), 0)\}$$









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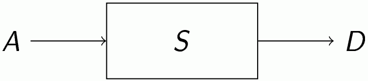
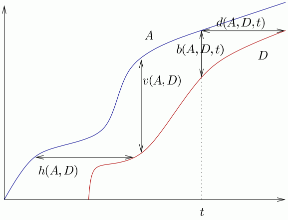
Flows and servers





- Flow : Cumulative curve A
  - $A(t)$: amount of data sent up to time t
  - Properties: null at 0 (and before), non decreasing
- Server: simple input/output relation: $S \subset \mathcal{F}_0 \times \mathcal{F}_0$
  - Property: departure/output produced after arrival/input:

    $A \xrightarrow{S} D \implies A \geq D$
- Worst delay $d(A, S)$ and buffer use $b(A, S)$.

$$d(A, S) = \sup_{A \xrightarrow{S} D} h(A, D) \qquad b(A, S) = \sup_{A \xrightarrow{S} D} v(A, D) \qquad (1)$$

Arrival and service curves

- A, S, D are real behaviours
  - unknown at design time

    $\Longrightarrow$ use of contracts
- Traffic contract: *arrival curve*

  A flow A has arrival curve α ($A \preceq \alpha$) iff

$$\forall t, d \geq 0 : A(t+d) - A(t) \leq \alpha(d) \iff A \leq A * \alpha \qquad (2)$$

- Server contract: *simple service curve*

  A server S offers a minimal service of curve β ($S \trianglerighteq \beta$) iff

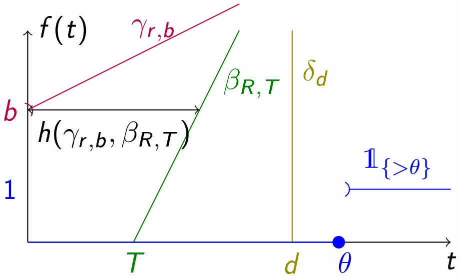
$$\forall A \xrightarrow{S} D : D \geq A * \beta \qquad (3)$$

- Server contract: *strict service curve*

  A server S offers a strict minimal service of curve β ($S \triangleright \beta$) iff on any backlogged period $(s, t]$ (*i.e.* $\forall x \in (s,t], A(x) > D(x)$)

$$D(t) - D(s) \geq \beta(t-s) \qquad (4)$$

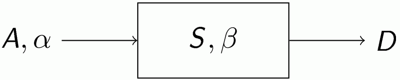
Common curves



$$\gamma_{r,b}(t) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{t=0} \\ rt + b & \text{t>0} \end{cases} \qquad \beta_{R,T}(t) \stackrel{\text{def}}{=} \begin{cases} 0 & t < T \\ Rt - T & t \geq T \end{cases}$$

$$\delta_d(t) \stackrel{\text{def}}{=} \begin{cases} 0 & t < d \\ \infty & t \geq T \end{cases} \qquad \mathbb{1}_{\{>\theta\}}(t) \stackrel{\text{def}}{=} \begin{cases} 0 & t \leq \theta \\ 1 & t > \theta \end{cases}$$

Network calculus: single flow results



- From arrival curve α and service curve β, bounds can be computed:

$$A \preceq \alpha, S \trianglerighteq \beta \implies d(A, S) \leq h(\alpha, \beta) \tag{5}$$

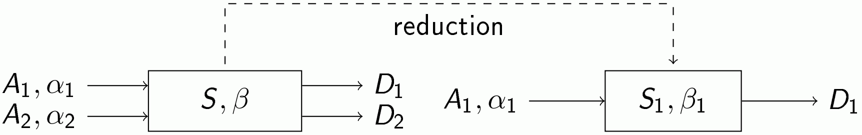
- A strict service β is a simple service β

$$S \triangleright \beta \implies S \trianglerighteq \beta$$

- If S offers a strict service β, it also offers $[\beta]^+_\uparrow$

$$S \triangleright \beta \implies S \triangleright [\beta]^+_\uparrow$$

Network calculus: aggregated flow results



- Blind multiplexing: assume strict service

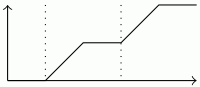
$$\beta_1 = [\beta - \alpha_2]^+_\uparrow \tag{6}$$

- FIFO multiplexing: assume simple service, chose some $\theta \geq 0$,

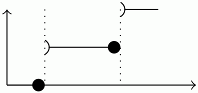
$$\beta_1^\theta = [\beta - \alpha_2 * \delta_\theta]^+ \mathbb{1}_{\{>\theta\}} \tag{7}$$

Amount of data, "up to time t"

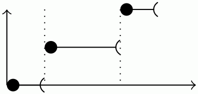
$A(t)$ is amount of data sent up to time t



(a) Continuous



(b) Left-cont.



(c) Right-cont.

- How to model "instantaneous" arrival of packet at time t?

Arrival "just after"	Arrival "at time t"
left-continuous	right-continuous
good mathematical property	more "intuitive" model

- Previous studies:
  - all assume left-continuous arrival curves
  - except the one handling packets

Does both modelling model same performances ?

- Are values of delay and memory usage the same?
- Yes: delays and backlog are not affected by continuity

$$v(A_l, D_l) = v(A_r, D_r) \qquad h(A_l, D_l) = h(A_r, D_r)$$

with f_l (resp. f_r) the left-continuous (resp. right-continuous) closure of f

Redo classical proofs with right-continuous assumption

	Left-cont.		Right-cont.
Blind multiplexing	$[\beta - \alpha_i]_{\uparrow}^{+}$	$\geq$	$[\beta - (\alpha_i)_r]_{\uparrow}^{+}$
FIFO multiplexing	$[\beta - \alpha_j * \delta_\theta]^{+} \mathbb{1}_{\{>\theta\}}$	$\leq$	$[\beta - \alpha_j * \delta_\theta]^{+} \mathbb{1}_{\{\geq\theta\}}$
Strict closure	idem $([\beta]_{\uparrow}^{+})$		
Strict is simple	idem		
Other details	see the paper		

- Main results are preserved
- Result expression changes
    - minor changes (continuity points)
    - no numerical impact (delays and backlog are not affected by continuity)

Conclusion

- Inconsistency of previous studies
- Formal study on continuity
  - no impact on delay and memory values
  - no impact on main properties
  - limited impact on expressions
  - no impact on delay and memory bounds